Topic: Horizontal and vertical tangent lines and differentiability

Question: Find the vertical tangent line(s) of the function.

$$f(x) = \frac{x+1}{x^2}$$

$$-4 \leq x \leq 5$$

Answer choices:

A $x = \frac{6}{25}$

B $x = \pm 1$

C $x = 0$

D $x = -\frac{3}{16}$

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Solution: C

The vertical tangent lines of a function exist where the derivative of the function does not exist or is undefined. In the case of a fraction, the derivative does not exist where the denominator of the fraction equals zero. Remember, if an interval is given, only the vertical tangent lines that fall inside the interval are considered and any other answers can be discarded.

Let's start with our original function

$$f(x) = \frac{x+1}{x^2}$$

Now let's use quotient rule to take the derivative.

$$f'(x) = \frac{(1)\left(x^2\right) - (x+1)(2x)}{\left(x^2\right)^2}$$

$$f'(x) = \frac{x^2 - 2x^2 - 2x}{x^4}$$

$$f'(x) = \frac{-x^2 - 2x}{x^4}$$

$$f'(x) = \frac{-x - 2}{x^3}$$

Remember, vertical tangent lines of a function exist where the derivative of the function does not exist. In this case, that happens when the denominator of our derivative equals 0. So setting the denominator equal to 0, we get

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$$x^3 = 0$$

$$x = 0$$

The function

$$f(x) = \frac{x+1}{x^2}$$

on the interval $-4 \leq x \leq 5$ has one vertical tangent line at $x = 0$.

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Topic: Horizontal and vertical tangent lines and differentiability

Question: Say where the function is differentiable on the interval.

$$f(x) = x\sqrt{4 - x^2}$$

$$-2 \leq x \leq 2$$

Answer choices:

A Not differentiable when $x = 0$.

B Differentiable for all values of x.

C Differentiable for all values of y.

D Not differentiable when $x = \pm 2$.

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Solution: D

A function is differentiable everywhere that the derivative exists. Since the derivative won't exist at vertical tangent lines, we know that the function is not differentiable at vertical tangent lines.

In the case of a fraction, the derivative does not exist where the denominator of the fraction equals 0. Remember, if an interval is given, only the vertical tangent lines that fall inside the interval are considered and any other answers can be discarded.

Let's start with our original function

$$f(x) = x\sqrt{4 - x^2}$$

Now, let's use product rule to take the derivative.

$$f'(x) = (1)\left(\sqrt{4 - x^2}\right) + (x)\left[\frac{1}{2}\left(4 - x^2\right)^{-\frac{1}{2}}(-2x)\right]$$

$$f'(x) = \sqrt{4 - x^2} - x^2\left(4 - x^2\right)^{-\frac{1}{2}}$$

$$f'(x) = \sqrt{4 - x^2} - \frac{x^2}{\left(4 - x^2\right)^{\frac{1}{2}}}$$

$$f'(x) = \sqrt{4 - x^2} - \frac{x^2}{\sqrt{4 - x^2}}$$

$$f'(x) = \frac{4 - x^2 - x^2}{\sqrt{4 - x^2}}$$

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$$f'(x) = \frac{4 - 2x^2}{\sqrt{4 - x^2}}$$

Remember, vertical tangent lines of a function exist where the derivative of the function does not exist which in this case occurs when the denominator of our fraction equals 0. So setting the denominator equal to 0, we get

$$\sqrt{4 - x^2} = 0$$

$$4 - x^2 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

The function $f(x) = x\sqrt{4 - x^2}$ on the interval $-2 \leq x \leq 2$ is not differentiable at $x = \pm 2$.

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Topic: Horizontal and vertical tangent lines and differentiability

Question: Where is the function differentiable on the interval?

$$f(x) = \frac{1}{x}$$

on the interval $-1 \leq x \leq 3$

Answer choices:

A $-1 \leq x \leq 1 \cup 1 \leq x \leq 3$

B $-1 \leq x < 1 \cup 1 < x \leq 3$

C $-1 \leq x \leq 0 \cup 0 \leq x \leq 3$

D $-1 \leq x < 0 \cup 0 < x \leq 3$

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Solution: D

A function is differentiable on an interval when it is continuous, with no vertical tangent lines.

We can see that the given function

$$f(x) = \frac{1}{x}$$

is discontinuous and undefined at $x = 0$.

To find vertical tangent lines of the function, we'll find the derivative.

$$f(x) = x^{-1}$$

$$f'(x) = -x^{-2}$$

$$f'(x) = -\frac{1}{x^2}$$

Vertical tangent lines will exist where the derivative is undefined. This derivative is undefined where $x = 0$. This means that the function is differentiable everywhere in the interval except at $x = 0$. The function is continuous everywhere else on the interval except for this one point, so we can write the differentiability of the interval as $-1 \leq x < 0 \cup 0 < x \leq 3$.

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